

# Disentangling the Spin-Parity of a Resonance via the Gold-Plated Decay Mode

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Searching for new resonances and finding out their properties is an essential part of any existing or future particle physics experiment. The nature of a new resonance is characterized by its spin, charge conjugation, parity, and its couplings with the existing particles of the Standard Model. If a new resonance is found in the four lepton final state produced via two intermediate  $Z$  bosons, the resonance could be a new heavy scalar or a  $Z'$  boson or even a higher spin particle. In such cases the step by step methodology as enunciated in this paper can be followed to determine the spin, parity and the coupling to two  $Z$  bosons of the parent particles, in a fully model-independent way. In our approach we show how three uniaxial distributions and few experimentally measurable observables can conclusively tell us about the spin, parity as well as the couplings of the new resonance to two  $Z$  bosons. We have performed a numerical analysis to validate our approach and showed how the uniaxial observables can be used to disentangle the spin parity as well as coupling of the resonance.

## I. INTRODUCTION

With the recent discovery of the ‘Higgs’ boson [1, 2], all the ingredients of the standard model of particle physics (SM) have been found. However, we do know that the SM does not fully explain the whole of the nature at its most fundamental level. For example, the problem of naturalness, the existence of extremely small masses for the neutrinos required to explain the observed neutrino oscillations, the abundance of matter over anti-matter in our observable universe, the constituents of dark matter (which is about five times more abundant than the ordinary matter) are a few of many issues which can not be handled in SM. So SM encompasses an incomplete description of nature and hence it must be supplemented or extended with some other hitherto unknown new physics. Any model of new physics invariably includes new interactions and thus many new particles. In order to have a comprehensive view of new physics it is therefore essential to look for new fundamental particles in experiments such as the Large Hadron Collider (LHC) or in the proposed future experiments such as the International Linear Collider (ILC). It is however important to have model independent methods at place to characterize resonances that would be observed in these high luminosity and high energy experiments. This paper is a step forward in that direction.

Similar idea was espoused in Ref. [3] in connection with the 125-126 GeV ‘Higgs’ resonance. Since the ‘Higgs’ was found to decay to two photons, the spin-1 possibility in this case was ruled out using Landau-Yang theorem. This paper, however, has the complete theoretical analysis and we do consider the spin-1 possibility here. The decay of a resonance to four leptons via a pair of  $Z$  bosons is considered as the gold

plated mode for discovery of any resonance in a hadron collider such as LHC. This mode can be used to search for many new resonances, such as a heavy scalar resonance, a  $Z'$  boson or a Kaluza-Klein boson or a massive graviton. Hence, all the spin possibilities must be considered. We analyse the decay of a resonance (say,  $X$  with spin  $J$ ) via this golden channel:  $X \rightarrow Z^{(*)}Z^{(*)} \rightarrow (\ell_1^-\ell_1^+)(\ell_2^-\ell_2^+)$ , where  $\ell_1, \ell_2$  are leptons  $e$  or  $\mu$ . Since we are considering an unknown particle with unknown mass, it may be heavy enough to produce two real  $Z$  bosons or we can have one on-shell  $Z$  and another off-shell  $Z$ , or both the  $Z$ ’s can be off-shell. We emphasize that the final state  $(e^+e^-)(\mu^+\mu^-)$  is not equivalent to  $(e^+e^-)(e^+e^-)$  or  $(\mu^+\mu^-)(\mu^+\mu^-)$  as sometimes mentioned in the literature, since the latter final states have to be anti-symmetrized with respect to each of the two sets of identical fermions in the final state. The anti-symmetrization of the amplitudes is not done in our analysis and hence our analysis applies only to  $(e^+e^-)(\mu^+\mu^-)$ . Since  $X$  decays to two  $Z$  bosons which are vector bosons,  $X$  can have spin possibilities  $J = 0, 1, 2$ . Higher spin possibilities ( $J \geq 3$ ) need not be considered as the number of independent helicity amplitudes remains three (for higher odd integer spins) or six (for higher even integer spins), and the only change in the amplitude comes from extra powers of momentum of the  $Z$  bosons [4]. This was shown by an example in Ref. [3]. In this paper we examine the angular distributions for the different spins  $J = 0, 1, 2$  and parity of such a new resonance  $X$  and present a strategy to determine them, as well as the couplings of  $X$  to the pair of  $Z$  bosons with the help of a few well defined experimental observables.

We start by considering the most general effective Lagrangian for each spin possibility of  $X$  decaying to two  $Z$  bosons and then write down the corresponding decay vertices. We evaluate the partial decay rate of  $X$  in terms of the invariant mass squared of the dileptons produced from the two  $Z$  decays and the angular distributions of the four lepton final state. We demonstrate that by studying three uni-angular distributions (i.e. *distributions involving only one angle*) one can almost completely determine the spin and parity of  $X$  and also explore any anomalous couplings in the most general fashion.

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We provide some experimental observables that can be tested to predict whether the resonance is a parity eigenstate or not irrespective of its spin. Then we analyse the nuances of differences in the uni-angular distributions which we take into consideration for separating each of the spin, parity possibilities. For this we express our uni-angular distributions in terms of helicity amplitudes in the transversity basis which are very effective in their sensitivity to the parity of the parent particle. We find that the  $J = 1$  possibility can be easily distinguished from the  $J = 0, 2$  possibilities since the uni-angular distributions for  $J = 1$  are completely predictable.

Nelson [5–7] and Dell’Aquila [6] realized the significance of studying angular correlations in such processes as Higgs ( $J = 0$ ) boson decaying to a pair of  $Z$  bosons for inferring the nature of the Higgs boson. Refs. [8–11] were first to extend the analysis to include higher spin possibilities. We study similar angular correlations in this paper to unambiguously determine the spin and parity of any new resonance decaying to four leptons via two  $Z$  boson. We begin the study by considering the most general  $XZZ$  vertices for  $J = 0$ ,  $J = 1$  and  $J = 2$  resonance  $X$ . Then we find out the effective Lagrangians and uniangular distributions in terms of different observables for each spin possibilities and layout a procedure to identify them. We emphasize that the full angular distribution for the decay is described in terms of orthogonal function of sine and cosine of the angles involved in the decay. Hence, no information is lost by considering uniangular distributions.

Decay of a resonance into two  $Z$  bosons which then decay into four leptons has been well studied in literature for different spin possibilities. For example Refs. [12–22] as well as Refs. [8–11] discuss extensively coupling of  $X$  to two  $Z$  bosons for different spin possibilities as well as how to determine the spin-parity of a new resonance decaying into four leptons via a pair of  $Z$  bosons. More recent references for Higgs decay to four leptons via two  $Z$  bosons can be found in Refs. [3, 19, 23–25]. The aim of the current paper should not be confused with that of Ref. [3], which dealt only with the 125 GeV Higgs case. The current paper significantly improves upon the concepts presented in Ref. [3] by *generalizing them to search for new resonances of arbitrary mass and arbitrary spin-parity using the gold-plated decay mode*. The analysis now includes the possibility of a spin-1 resonance which can also decay to two on-shell  $Z$  bosons, a case which was not dealt with earlier. Ref. [17] does consider the spin-1 possibility. However, we emphasize that our analysis differs from this analysis as well, in many significant aspects. Our analysis does not depend on the resonance production mechanism. Unlike Ref. [17], we start with the most general effective Lagrangian for any spin and parity possibility. Our treatment in terms of transversity amplitudes is explicit on their parity behavior whereas the amplitudes of the other reference are ambivalent. We carefully use Schouten identity to show which specific combination of vertex factors contribute to the orthogonal transversity amplitudes.

We show numerically that the uniangular distributions can indeed be used to disentangle the spin and parity of the resonance  $X$ . We benchmark our analysis for the  $J = 1$  possibility of  $X$  and calculate the values of the angular asymmetries for

the 14 TeV and 33 TeV LHC. The analysis can also be extended for  $J = 2$  spin possibility.

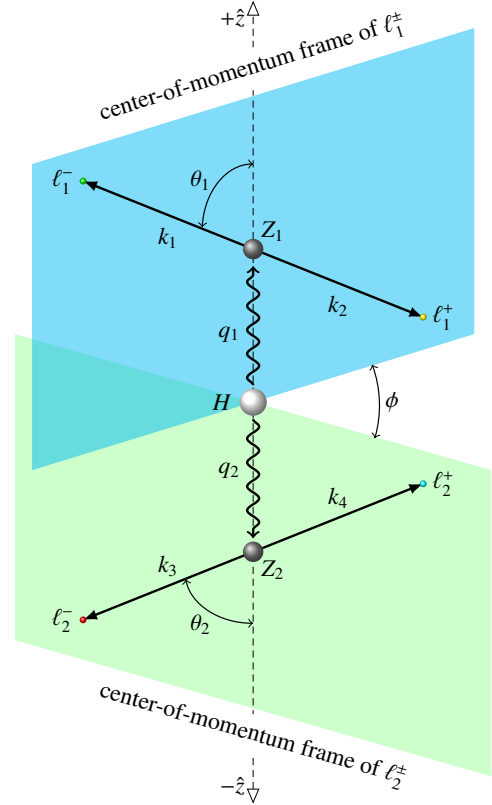


FIG. 1. Definition of the polar angles ( $\theta_1$  and  $\theta_2$ ) and the azimuthal angle ( $\phi$ ) in the decay of  $X$  to a pair of  $Z$ 's, and then to four charged leptons:  $X \rightarrow Z_1 + Z_2 \rightarrow (\ell_1^- + \ell_1^+) + (\ell_2^- + \ell_2^+)$ , where  $\ell_1, \ell_2 \in \{e, \mu\}$ . It should be clear from the figure that  $\vec{k}_1 = -\vec{k}_2$  and  $\vec{k}_3 = -\vec{k}_4$ . The lepton pairs are reconstructed in their respective center-of-momentum frames. The angle between the two decay planes is the angle  $\phi$ . The spin projection of particle  $X$  is along the  $\hat{z}$  direction.

A few words about the notation followed in this paper is noteworthy. Since our aim in this paper is to get an insight into the spin and parity of the resonance under consideration, the notation has been carefully designed in such a way that at any stage in the paper one would have no difficulty in recognizing the terms (in Lagrangian or vertex factor or amplitude) which contribute for even and odd parity cases. We use  $e_i^{(j)}$  ( $o_i^{(j)}$ ),  $E_i^{(j)}$  ( $O_i^{(j)}$ ), and  $A_{Ei}^{(j)}$  ( $A_{Oi}^{(j)}$ ) for parity-even (parity-odd) coupling constants, form factors and helicity amplitudes respectively for spin  $J$  case in general. All the notations are described when they are introduced in the paper.

Sec. II deals completely with the stepwise building of the theoretical analysis. In subsection II A we write down most general Lagrangian and corresponding vertex factor for each possible spin. Transversity amplitudes and uni-angular distributions for each of the spin possibilities are analyzed in subsections II B and II C respectively. In Sec. III we discuss about the possibility of finding one such resonance and investigate how to disentangle them. In subsection III A we find the uni-angular distributions and the values of the observables for a

spin  $1^+$  heavy resonance in future 14 TeV and 33 TeV LHC run. We repeat the analysis for a spin  $1^-$  resonance in subsection III B. We summarize in Sec. IV emphasizing the importance of our results.

## II. ANALYSIS OF THE DECAY OF A RESONANCE TO FOUR FINAL CHARGED LEPTONS VIA TWO Z BOSONS

Let us consider the decay of  $X$  to four charged leptons via a pair of  $Z$  bosons:

$$X \rightarrow Z_1 + Z_2 \rightarrow (\ell_1^- + \ell_1^+) + (\ell_2^- + \ell_2^+),$$

where  $\ell_1, \ell_2$  are leptons  $e$  or  $\mu$ . As mentioned in the introduction we assume  $\ell_1$  and  $\ell_2$  are not identical. The kinematics for the decay is as shown in Fig. 1. The resonance  $X$  at rest decays to two  $Z$  bosons (none of them or either of them or both of them can be on-shell depending on the mass of  $X$ )  $Z_1$  and  $Z_2$  moving along the  $+\hat{z}$  and  $-\hat{z}$  directions with four-momenta  $q_1$  and  $q_2$  respectively. The decays of  $Z_1$  and  $Z_2$  are considered in their respective rest frames. The angles and momenta involved are as described in Fig. 1. The four-momentum of  $X$  is defined as  $P$ :  $P = q_1 + q_2$ . We also define  $Q = q_1 - q_2$ . We choose  $Z_1$  to decay to lepton pair  $\ell_1^\pm$  with momenta  $k_1$  and  $k_2$  respectively and  $Z_2$  to decay to  $\ell_2^\pm$  with momenta  $k_3$  and  $k_4$  respectively. We define two unit normals  $\vec{n}_1$  and  $\vec{n}_2$  to the planes containing  $\vec{k}_1, \vec{k}_2$  and  $\vec{k}_3, \vec{k}_4$  respectively by

$$\vec{n}_1 = \frac{\vec{k}_1 \times \vec{k}_2}{|\vec{k}_1 \times \vec{k}_2|} = -\hat{y}, \quad (1)$$

$$\vec{n}_2 = \frac{\vec{k}_3 \times \vec{k}_4}{|\vec{k}_3 \times \vec{k}_4|} = -(\sin \phi \hat{x} - \cos \phi \hat{y}). \quad (2)$$

Thus the azimuthal angle  $\phi$  can be specified unambiguously by

$$\vec{n}_1 \cdot \vec{n}_2 = -\cos \phi, \quad (3)$$

$$\vec{n}_1 \times \vec{n}_2 = -\sin \phi \hat{z}. \quad (4)$$

We begin the study by considering the most general  $XZZ$  interaction Lagrangians for a  $J = 0$ , a  $J = 1$  and a  $J = 2$  resonance  $X$ . From the Lagrangians we find out the most general  $XZZ$  decay vertex factor for each spin possibility.

### A. Most general interaction Lagrangian and Vertex factor for the decay $X \rightarrow ZZ$

Considering Lorentz invariance one can write down the most general interaction Lagrangian for the decay of  $X$  to two  $Z$  bosons. The Lagrangian depends on the spin of the parent resonance  $X$ . We denote the most general Lagrangian for spin- $J$  resonance  $X$  decaying to two  $Z$  bosons by  $\mathcal{L}_{XZZ}^{(J)}$ . The Lagrangians for the allowed spin possibilities are given by

- $J = 0$ :

$$\mathcal{L}_{XZZ}^{(0)} = e_1^{(0)} X^{(0)} Z^\alpha Z^\beta g_{\alpha\beta} + e_2^{(0)} X^{(0)} Z^{\mu\nu} Z_{\mu\nu}$$

$$+ i o_1^{(0)} X^{(0)} \tilde{Z}^{\mu\nu} Z_{\mu\nu}, \quad (5)$$

- $J = 1$ :

$$\begin{aligned} \mathcal{L}_{XZZ}^{(1)} = & o_1^{(1)} X_\mu^{(1)} Z^{\mu\nu} Z_\nu + o_2^{(1)} X_{\mu\nu}^{(1)} Z^\mu Z^\nu \\ & + i e_1^{(1)} X_\mu^{(1)} \tilde{Z}_{\mu\nu} Z^\nu + i e_2^{(1)} \tilde{X}_{\mu\nu}^{(1)} Z^\mu Z^\nu, \end{aligned} \quad (6)$$

- $J = 2$ :

$$\begin{aligned} \mathcal{L}_{XZZ}^{(2)} = & e_1^{(2)} X_{\mu\nu}^{(2)} Z^\mu Z^\nu + e_2^{(2)} X_{\mu\nu}^{(2)} Z^{\mu\alpha} Z^{\nu\beta} g_{\alpha\beta} \\ & + e_3^{(2)} X_{\mu\nu}^{(2)} (\partial_\beta Z^{\mu\alpha}) (\partial_\alpha Z^{\nu\beta}) \\ & + e_4^{(2)} (\partial_\alpha \partial_\beta X_{\mu\nu}^{(2)}) (Z^{\mu\alpha} Z^{\nu\beta}) \\ & + e_5^{(2)} X_{\mu\nu}^{(2)} (\partial^\mu Z^{\alpha\beta}) (\partial^\nu Z_{\alpha\beta}) \\ & + i o_1^{(2)} X_{\mu\nu}^{(2)} \tilde{Z}^{\mu\alpha} Z^{\nu\beta} g_{\alpha\beta} \\ & + i o_2^{(2)} X_{\mu\nu}^{(2)} (\partial_\beta \tilde{Z}^{\mu\alpha}) (\partial_\alpha Z^{\nu\beta}) \\ & + i o_3^{(2)} (\partial_\alpha \partial_\beta X_{\mu\nu}^{(2)}) (\tilde{Z}^{\mu\alpha} Z^{\nu\beta}) \\ & + i o_4^{(2)} X_{\mu\nu}^{(2)} (\partial^\mu \tilde{Z}^{\alpha\beta}) (\partial^\nu Z_{\alpha\beta}), \end{aligned} \quad (7)$$

where  $e_i^{(J)}$  and  $o_i^{(J)}$  are the effective coupling constants for a specified spin  $J$  of the parent particle that come with the parity-even and parity-odd parts of the Lagrangian respectively;  $X^{(0)}, X_\mu^{(1)}, X_{\mu\nu}^{(2)}$  are the scalar, vector and tensor fields for the corresponding spin possibilities of the particle  $X$ ; the tensor  $X_{\mu\nu}^{(2)}$  is a symmetric, traceless and divergenceless tensor;  $Z^\alpha$  is the vector field for the  $Z$  boson; and  $X_{\mu\nu}^{(1)}, \tilde{X}_{\mu\nu}^{(1)}, Z_{\mu\nu}, \tilde{Z}_{\mu\nu}$  are defined as

$$X_{\mu\nu}^{(1)} = \partial_\mu X_\nu^{(1)} - \partial_\nu X_\mu^{(1)}, \quad (8)$$

$$\tilde{X}_{\mu\nu}^{(1)} = \epsilon_{\mu\nu\rho\sigma} X^{(1)\rho\sigma}, \quad (9)$$

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \quad (10)$$

$$\tilde{Z}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} Z^{\rho\sigma}. \quad (11)$$

It is noteworthy to clearly emphasize that the two quantities  $X_{\mu\nu}^{(1)}$  and  $X_{\mu\nu}^{(2)}$  must not be confused. The quantity  $X_{\mu\nu}^{(1)}$  is anti-symmetric under the exchange of  $\mu \leftrightarrow \nu$ , but  $X_{\mu\nu}^{(2)}$  is symmetric under the same exchange. Given the most general effective Lagrangian for each of the spin possibilities, it is imperative that one finds out the effective vertex factors from it. Using the effective vertex factors one then proceeds to find out the angular distributions for each of the spin possibilities, which are finally used to distinguish the different cases of spin and parity.

Analyzing the effective Lagrangians given in Eqs. (5), (6) and (7) we can write down the following vertex factors for each of the spin possibilities of  $X$ :

- $J = 0$ :

$$V_{XZZ}^{\alpha\beta} = E_1^{(0)} g^{\alpha\beta} + E_2^{(0)} P^\alpha P^\beta + i O_1^{(0)} \epsilon^{\alpha\beta\rho\sigma} q_{1\rho} q_{2\sigma}, \quad (12)$$

- $J = 1$ :

$$V_{XZZ}^{\mu;\alpha\beta} = O_1^{(1)} (g^{\alpha\mu} q_1^\beta + g^{\beta\mu} q_2^\alpha) + i E_1^{(1)} \epsilon^{\alpha\beta\mu\nu} Q_\nu, \quad (13)$$

•  $J = 2$ :

$$\begin{aligned}
V_{XZZ}^{\mu\nu;\alpha\beta} = & E_1^{(2)} (g^{\alpha\nu} g^{\beta\mu} + g^{\alpha\mu} g^{\beta\nu}) \\
& + E_2^{(2)} (Q^\mu (Q^\alpha g^{\beta\nu} + Q^\beta g^{\alpha\nu}) \\
& \quad + Q^\nu (Q^\alpha g^{\beta\mu} + Q^\beta g^{\alpha\mu})) \\
& + E_3^{(2)} (Q^\mu Q^\nu g^{\alpha\beta}) - E_4^{(2)} (Q^\alpha Q^\beta Q^\mu Q^\nu) \\
& + 2i O_1^{(2)} (g^{\beta\nu} \epsilon^{\alpha\mu\rho\sigma} - g^{\alpha\nu} \epsilon^{\beta\mu\rho\sigma} \\
& \quad + g^{\beta\mu} \epsilon^{\alpha\nu\rho\sigma} - g^{\alpha\mu} \epsilon^{\beta\nu\rho\sigma}) q_{1\rho} q_{2\sigma} \\
& + i O_2^{(2)} (Q^\beta (Q^\nu \epsilon^{\alpha\mu\rho\sigma} + Q^\mu \epsilon^{\alpha\nu\rho\sigma}) \\
& \quad - Q^\alpha (Q^\nu \epsilon^{\beta\mu\rho\sigma} + Q^\mu \epsilon^{\beta\nu\rho\sigma})) q_{1\rho} q_{2\sigma} \\
& + i O_3^{(2)} (\epsilon^{\alpha\beta\nu\rho} P_\rho Q^\mu + \epsilon^{\alpha\beta\mu\rho} P_\rho Q^\nu) \\
& + i O_4^{(2)} \epsilon^{\alpha\beta\rho\sigma} Q^\mu Q^\nu q_{1\rho} q_{2\sigma}, \tag{14}
\end{aligned}$$

where  $E_i^{(J)}$ ,  $O_i^{(J)}$  are the form factors which are associated with terms that are even and odd under parity respectively, and are related to the effective coupling constants  $e_i^{(J)}$ ,  $o_i^{(J)}$  (as given in A). It is important to notice that the terms proportional to the Levi-Civita tensor  $\epsilon^{\alpha\beta\mu\nu}$  are the terms that are odd under parity in the vertex factors for spin 0 and 2, but not in case of spin 1. This can be easily explained by the fact that in case of spin 1 there are three polarization vectors that come into picture, which in association with the Levi-Civita tensor and the momentum  $Q$  give rise to a triple product. This triple product involves the three polarizations and is even under parity. Thus the term with Levi-Civita tensor carries the even form factor in case of spin 1 instead of the usual odd form factor. Moreover, the imaginary  $i$  remains with the Levi-Civita tensor to keep the vertex factor even under time reversal.

It is noteworthy that in Ref. [3], the notation used is slightly different from the ones used here. In order to go to the notation used in Ref [3], the necessary substitutions are tabulated in Table I.

Spin 0	Spin 2
$E_1^{(0)} \rightarrow \frac{igM_Z}{\cos\theta_W} a$	$E_1^{(2)} \rightarrow A$
$E_2^{(0)} \rightarrow \frac{igM_Z}{\cos\theta_W} b$	$E_2^{(2)} \rightarrow B$
$O_1^{(0)} \rightarrow \frac{igM_Z}{\cos\theta_W} c$	$E_3^{(2)} \rightarrow C$
	$E_4^{(2)} \rightarrow D$
	$O_1^{(2)} \rightarrow E$
	$O_2^{(2)} \rightarrow F$
	$O_3^{(2)} \rightarrow G$

TABLE I. The correspondence between the vertex factors of this paper with those given in Ref. [3].

The Lagrangians and vertex factors for the different spin possibilities have also been considered in the literature before [8–11, 20, 26–29]. Our approach differs from these works by isolating the parity-even and odd terms explicitly, such that

the uniangular distributions can be profitably used for characterizing the spin and parity of the particle. It is important to notice that there is no contribution from  $o_2^{(1)}$  and  $e_2^{(1)}$  to the vertex factors. In order to understand this let us have a look at the terms  $X_{\mu\nu}^{(1)}$  and  $\tilde{X}_{\mu\nu}^{(1)}$  which come in association with  $o_2^{(1)}$  and  $e_2^{(1)}$  respectively. Under  $\mu \leftrightarrow \nu$  exchange both  $X_{\mu\nu}^{(1)}$  and  $\tilde{X}_{\mu\nu}^{(1)}$  pick up a relative negative sign. However, the  $Z^\mu Z^\nu$  part is symmetric under the  $\mu \leftrightarrow \nu$  exchange, as it should be for the case of two identical Z bosons. So considering Bose symmetry of the two daughter Z bosons, it is clear that the terms  $o_2^{(1)}$  and  $e_2^{(1)}$  cannot contribute, as these are overall anti-symmetric under Bose symmetry.

Before we embark upon the journey to find out the differential decay rate or the uni-angular distributions, it would be nice to have some physical understanding of the process under consideration. Analysing the decay  $X \rightarrow ZZ$  in terms of the helicity amplitudes and partial waves offer valuable insight to the understanding of the process.

## B. Helicity amplitudes in the transversity basis for the decay $X \rightarrow ZZ$

Helicity amplitudes in the transversity basis in which we are going to work have a very special property: the helicity amplitudes are now sensitive to the parity of the resonance  $X$ . Thus our helicity amplitudes have distinct and unambiguous parity signatures. We shall denote those helicity amplitudes which have parity-even form factors as  $A_{Ei}^{(J)}$  and those with parity-odd form factors as  $A_{Oi}^{(J)}$ , where the superscript ( $J$ ) denotes the spin of the parent particle  $X$ . The formalism for the helicity amplitudes is discussed in the B. Ignoring the lepton masses in the final state, we get the following helicity amplitudes for the different spin possibilities of  $X$ :

•  $J = 0$ :

$$A_{E1}^{(0)} = \frac{1}{2} (M_X^2 - M_1^2 - M_2^2) E_1^{(0)} + M_X^2 Y^2 E_2^{(0)}, \tag{15a}$$

$$A_{E2}^{(0)} = \sqrt{2} M_1 M_2 E_1^{(0)}, \tag{15b}$$

$$A_{O1}^{(0)} = \sqrt{2} M_1 M_2 M_X Y O_1^{(0)}, \tag{15c}$$

•  $J = 1$ :

$$A_{O1}^{(1)} = \frac{\sqrt{2}}{3} D_1 O_1^{(1)}, \tag{16a}$$

$$A_{E1}^{(1)} = \frac{1}{3} D_2 E_1^{(1)}, \tag{16b}$$

•  $J = 2$ :

$$\begin{aligned}
A_{E1}^{(2)} = & \frac{2\sqrt{2}}{3\sqrt{3}M_X^2} (E_1^{(2)} (M_X^4 - M_-^4) - E_2^{(2)} (8M_X^4 Y^2) \\
& + E_3^{(2)} (4M_X^2 Y^2) (M_+^2 - M_X^2) \\
& - E_4^{(2)} (8M_X^4 Y^4)), \tag{17a}
\end{aligned}$$

$$A_{E2}^{(2)} = \frac{8M_1 M_2}{3\sqrt{3}} (E_1^{(2)} + 4Y^2 E_3^{(2)}), \tag{17b}$$

$$A_{E3}^{(2)} = \frac{4}{3M_X M_+} \left( E_1^{(2)} (M_-^4 - M_X^2 M_+^2) + E_2^{(2)} (4M_+^2 M_X^2 Y^2) \right), \quad (17c)$$

$$A_{E4}^{(2)} = \frac{8M_1 M_2 \nu^2}{3M_X M_+} E_1^{(2)}, \quad (17d)$$

$$A_{O1}^{(2)} = \frac{4Y}{3M_+} \left( O_1^{(2)} (M_-^4 - M_X^2 M_+^2) + O_2^{(2)} (4M_+^2 M_X^2 Y^2) \right), \quad (17e)$$

$$A_{O2}^{(2)} = \frac{8M_1 M_2 \mu^2 Y}{3\sqrt{3}M_+} O_1^{(2)}, \quad (17f)$$

where  $M_X$  is the mass of the resonance  $X$ ,  $M_1$  is the invariant mass of the  $\ell_1^\pm$  lepton pair,  $M_2$  is the invariant mass of the  $\ell_2^\pm$  lepton pair (for off-shell contributions  $M_1^2, M_2^2$  are not equal to  $M_Z^2$ ),  $Y$  is the magnitude of the three-momentum with which the two  $Z$  bosons fly away back-to-back in the rest frame of  $X$ :

$$Y = \frac{\sqrt{\lambda(M_X^2, M_1^2, M_2^2)}}{2M_X}, \quad (18)$$

with

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx), \quad (19)$$

being the Källén function,  $D_1$  and  $D_2$  being given by

$$D_1^2 = 2M_X^6 M_+^2 - \frac{1}{2} M_X^4 (5M_+^4 + M_-^4) + 6M_X^2 (M_+^6 - M_+^2 M_-^4) + \frac{3}{2} M_+^4 M_-^4 - \frac{1}{2} M_-^8, \quad (20)$$

$$D_2^2 = 16M_X^6 M_+^2 + 56M_X^4 M_+^4 - 86M_X^2 M_+^6 + M_-^4 (-85M_X^4 + 96M_X^2 M_+^2 - 35M_+^4) + 38M_-^8, \quad (21)$$

with  $M_+, M_-, \mu^2$  and  $\nu^2$  being given by

$$M_+^2 = M_1^2 + M_2^2, \quad (22)$$

$$M_-^2 = M_1^2 - M_2^2, \quad (23)$$

$$\mu^4 = 4M_X^2 M_+^2 + 3M_-^4, \quad (24)$$

$$\nu^4 = 2M_X^2 M_+^2 + M_-^4. \quad (25)$$

It is noteworthy that considering the vertex factors with form factors  $E_1^{(2)}, E_2^{(2)}, E_3^{(2)}, E_4^{(2)}, O_1^{(2)}$  and  $O_2^{(2)}$  suffice to provide the most general angular distribution for the spin-2 case. Including the form factors  $O_3^{(2)}$  and  $O_4^{(2)}$  result in only a redefinition of the form factors  $O_1^{(2)}$  and  $O_2^{(2)}$  as discussed in C.

It is again noteworthy that the amplitudes used here are notationally different from the amplitudes used in Ref. [3] for better clarity of their physical content. By following the substitutions as given in Table II one can go back to the notations used in Ref. [3]. The notations  $M_\pm, \mu$  and  $\nu$  have been introduced to make the power counting of mass dimensions easy.

### C. Uni-angular distributions for the decay

$$X \rightarrow Z_1 Z_2 \rightarrow (\ell_1^+ \ell_1^-)(\ell_2^+ \ell_2^-)$$

Using the helicity amplitudes defined in Eqs. (15), (16) and (17), the uni-angular distributions w.r.t  $\cos \theta_1, \cos \theta_2$  and  $\phi$

Spin 0	Spin 2
$A_{E1}^{(0)} \rightarrow A_L$	$A_{O1}^{(2)} \rightarrow A_L$
$A_{E2}^{(0)} \rightarrow A_\parallel$	$A_{O2}^{(2)} \rightarrow A_M$
$A_{O1}^{(0)} \rightarrow A_\perp$	$A_{E1}^{(2)} \rightarrow A_1$
	$A_{E2}^{(2)} \rightarrow A_2$
	$A_{E3}^{(2)} \rightarrow A_3$
	$A_{E4}^{(2)} \rightarrow A_4$
	$Y \rightarrow X$
	$M_+ \rightarrow u_1$
	$M_- \rightarrow u_2$
	$\mu^2 \rightarrow \nu$
	$\nu^2 \rightarrow w$

TABLE II. The correspondence between the helicity amplitudes of this paper and those given in Ref. [3].

can be written in a unified notation for all the allowed spin possibilities of  $X$  as follows:

$$\frac{1}{\Gamma_f^{(j)}} \frac{d^3 \Gamma^{(j)}}{dq_1^2 dq_2^2 d \cos \theta_1} = \frac{1}{2} + T_1^{(j)} \cos \theta_1 + T_2^{(j)} P_2(\cos \theta_1), \quad (26)$$

$$\frac{1}{\Gamma_f^{(j)}} \frac{d^3 \Gamma^{(j)}}{dq_1^2 dq_2^2 d \cos \theta_2} = \frac{1}{2} + T_1'^{(j)} \cos \theta_2 + T_2'^{(j)} P_2(\cos \theta_2), \quad (27)$$

$$\frac{2\pi}{\Gamma_f^{(j)}} \frac{d^3 \Gamma^{(j)}}{dq_1^2 dq_2^2 d\phi} = 1 + U_1^{(j)} \cos \phi + U_2^{(j)} \cos 2\phi + V_1^{(j)} \sin \phi + V_2^{(j)} \sin 2\phi, \quad (28)$$

where  $T_i^{(j)}, T_i'^{(j)}, U_i^{(j)}$ , and  $V_i^{(j)}$  are the coefficients of  $P_i(\cos \theta_1), P_i(\cos \theta_2), \cos(i\phi)$ , and  $\sin(i\phi)$  respectively with  $P_i(x)$  being the  $i$ 'th Legendre polynomial in  $x$ . These coefficients can easily be obtained from the uni-angular distributions by means of the following asymmetries:

$$T_1^{(j)} = \left( -\int_{-1}^0 + \int_0^{+1} \right) d \cos \theta_1 \left( \frac{1}{\Gamma_f^{(j)}} \frac{d^3 \Gamma^{(j)}}{dq_1^2 dq_2^2 d \cos \theta_1} \right), \quad (29)$$

$$T_2^{(j)} = \frac{4}{3} \left( \int_{-1}^{-\frac{1}{2}} - \int_{-\frac{1}{2}}^0 - \int_0^{+\frac{1}{2}} + \int_{+\frac{1}{2}}^{+1} \right) d \cos \theta_1 \times \left( \frac{1}{\Gamma_f^{(j)}} \frac{d^3 \Gamma^{(j)}}{dq_1^2 dq_2^2 d \cos \theta_1} \right), \quad (30)$$

$$T_1'^{(j)} = \left( -\int_{-1}^0 + \int_0^{+1} \right) d \cos \theta_2 \left( \frac{1}{\Gamma_f^{(j)}} \frac{d^3 \Gamma^{(j)}}{dq_1^2 dq_2^2 d \cos \theta_2} \right), \quad (31)$$

$$T_2'^{(j)} = \frac{4}{3} \left( \int_{-1}^{-\frac{1}{2}} - \int_{-\frac{1}{2}}^0 - \int_0^{+\frac{1}{2}} + \int_{+\frac{1}{2}}^{+1} \right) d \cos \theta_2 \times \left( \frac{1}{\Gamma_f^{(j)}} \frac{d^3 \Gamma^{(j)}}{dq_1^2 dq_2^2 d \cos \theta_2} \right), \quad (32)$$

$$U_1^{(j)} = \frac{1}{4} \left( -\int_{-\pi}^{-\frac{\pi}{2}} + \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} - \int_{+\frac{\pi}{2}}^{+\pi} \right) d\phi \left( \frac{2\pi}{\Gamma_f^{(j)}} \frac{d^3\Gamma^{(j)}}{dq_1^2 dq_2^2 d\phi} \right), \quad (33)$$

$$U_2^{(j)} = \frac{1}{4} \left( \int_{-\pi}^{-\frac{3\pi}{4}} - \int_{-\frac{3\pi}{4}}^{-\frac{\pi}{4}} + \int_{-\frac{\pi}{4}}^{+\frac{\pi}{4}} - \int_{+\frac{\pi}{4}}^{+\frac{3\pi}{4}} + \int_{+\frac{3\pi}{4}}^{+\pi} \right) d\phi \times \left( \frac{2\pi}{\Gamma_f^{(j)}} \frac{d^3\Gamma^{(j)}}{dq_1^2 dq_2^2 d\phi} \right), \quad (34)$$

$$V_1^{(j)} = \frac{1}{4} \left( -\int_{-\pi}^0 + \int_0^{+\pi} \right) d\phi \left( \frac{2\pi}{\Gamma_f^{(j)}} \frac{d^3\Gamma^{(j)}}{dq_1^2 dq_2^2 d\phi} \right), \quad (35)$$

$$V_2^{(j)} = \frac{1}{4} \left( \int_{-\pi}^{-\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^0 + \int_0^{+\frac{\pi}{2}} - \int_{+\frac{\pi}{2}}^{+\pi} \right) d\phi \times \left( \frac{2\pi}{\Gamma_f^{(j)}} \frac{d^3\Gamma^{(j)}}{dq_1^2 dq_2^2 d\phi} \right). \quad (36)$$

The differential decay width  $\Gamma_f^{(j)}$  is defined as

$$\Gamma_f^{(j)} = \frac{d^2\Gamma^{(j)}}{dq_1^2 dq_2^2} = \mathcal{N}^{(j)} \left( \sum_i |A_{Ei}^{(j)}|^2 + \sum_j |A_{Oj}^{(j)}|^2 \right) \quad (37)$$

where the factor  $\mathcal{N}^{(j)}$  is given by

$$\mathcal{N}^{(j)} = S^{(j)} \frac{9}{2^{10}} \frac{1}{\pi^4} \frac{\text{Br}_{\ell\ell}^2}{M_X^2} \frac{\Gamma_Z^2}{M_Z^2} Y \frac{1}{(q_1^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \times \frac{1}{(q_2^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}, \quad (38)$$

with  $S^{(j)}$  being the factor that comes from summing over initial spin states, i.e.  $S^{(j)} = 1, \frac{1}{3}, \frac{1}{5}$  for scalar, vector and tensor resonances respectively,  $\Gamma_Z$  being the total decay width of the  $Z$  boson,  $\text{Br}_{\ell\ell}$  being the branching ratio for the decay of  $Z$  to a pair of charged leptons:  $Z \rightarrow \ell^+ \ell^-$ . The coefficients  $T_i^{(j)}$ ,  $T_i'^{(j)}$ ,  $U_i^{(j)}$ ,  $V_i^{(j)}$  introduced in Eqs. (26), (27), (28) are expressed in terms of *helicity fractions*  $F_{Ei}^{(j)}$  and  $F_{Oj}^{(j)}$  which are defined as

$$\left. \begin{aligned} F_{Ei}^{(j)} &= \frac{A_{Ei}^{(j)}}{\sum_i |A_{Ei}^{(j)}|^2 + \sum_j |A_{Oj}^{(j)}|^2}, \\ F_{Oj}^{(j)} &= \frac{A_{Oj}^{(j)}}{\sum_i |A_{Ei}^{(j)}|^2 + \sum_j |A_{Oj}^{(j)}|^2}, \end{aligned} \right\} \quad (39)$$

such that

$$\sum_i |F_{Ei}^{(j)}|^2 + \sum_j |F_{Oj}^{(j)}|^2 = 1. \quad (40)$$

The expressions for the coefficients  $T_i^{(j)}$ ,  $T_i'^{(j)}$ ,  $U_i^{(j)}$ ,  $V_i^{(j)}$  are given in D. Looking at the uni-angular distributions it is easy to find that for any spin case, the coefficients  $T_1^{(j)}$ ,  $T_1'^{(j)}$ ,  $V_1^{(j)}$  and  $V_2^{(j)}$  are the interference terms between the parity-even and parity-odd terms. So if the resonance  $X$  were to be a parity eigenstate, these interference terms must vanish irrespective

of the spin of  $X$ . Therefore, the conditions for  $X$  to be a parity eigenstate are

$$T_1^{(j)} = T_1'^{(j)} = V_1^{(j)} = V_2^{(j)} = 0. \quad (41)$$

Now it is again easy to observe that for spin-0 case the coefficients  $T_2^{(0)}$  and  $T_2'^{(0)}$  are the same. However this is not true for spin-1 and spin-2 cases, in general. In order to further analyse this situation we define the difference  $\Delta^{(j)}$  between  $T_2^{(j)}$  and  $T_2'^{(j)}$ :

$$\Delta^{(j)} = T_2^{(j)} - T_2'^{(j)}. \quad (42)$$

For the different spin possibilities we have

$$\Delta^{(0)} = 0, \quad (43)$$

$$\Delta^{(1)} = 6M_X^2 M_-^2 Y^2 \left( \frac{|F_{O1}^{(1)}|^2 (M_X^2 + M_+^2)}{D_1^2} + \frac{2|F_{E1}^{(1)}|^2 (5M_X^2 + M_+^2)}{D_2^2} \right), \quad (44)$$

$$\Delta^{(2)} = \frac{3M_-^2}{4M_+^2 \mu^4 \nu^4} \left( \mu^4 \nu^4 (|F_{E3}^{(2)}|^2 + |F_{O1}^{(2)}|^2) - M_-^4 (\mu^4 |F_{E4}^{(2)}|^2 + 3\nu^4 |F_{O2}^{(2)}|^2) \right) + \frac{3M_1 M_2 M_-^2}{M_+^2 \mu^2 \nu^2} \left( \mu^2 \text{Re}(F_{E3}^{(2)} F_{E4}^{(2)*}) + \sqrt{3} \nu^2 \text{Re}(F_{O1}^{(2)} F_{O2}^{(2)*}) \right). \quad (45)$$

It is clear that we can never have  $\Delta^{(1)} = 0$  for all values of  $M_1$  and  $M_2$ , since this requires  $F_{E1}^{(1)} = F_{O1}^{(1)} = 0$ , which is an unphysical condition to satisfy as it totally obliterates the vertex factor itself. Requiring that  $\Delta^{(2)} = 0$  for all values of  $M_1$  and  $M_2$  implies that  $F_{E3}^{(2)} = F_{E4}^{(2)} = F_{O1}^{(2)} = F_{O2}^{(2)} = 0$ , which in turn implies that  $E_1^{(2)} = E_2^{(2)} = O_1^{(2)} = O_2^{(2)} = 0$ . Thus it leaves out two form factors in the vertex factor, namely  $E_3^{(2)}$  and  $E_4^{(2)}$ . This case constitutes a very special case of the spin 2 scenario and all its uni-angular distributions are completely indistinguishable from the corresponding angular distributions for a parity even spin-0 resonance. A closer look at the helicity amplitudes for these two cases,  $J^P = 0^+$  and special  $2^+$ , reveals that the special  $2^+$  case has extra  $Y^2$  dependence when compared with  $0^+$  case:

$$A_{E1}^{(0)} = \frac{1}{2} (M_X^2 - M_1^2 - M_2^2) E_1^{(0)} + M_X^2 Y^2 E_2^{(0)}, \quad (46)$$

$$A_{E2}^{(0)} = \sqrt{2} M_1 M_2 E_1^{(0)}, \quad (47)$$

$$A_{E1}^{(2)} = \left( -\frac{16\sqrt{2}}{3\sqrt{3}} Y^2 \right) \left( \frac{1}{2} (M_X^2 - M_1^2 - M_2^2) E_3^{(2)} + M_X^2 Y^2 E_4^{(2)} \right), \quad (48)$$

$$A_{E2}^{(2)} = \left( \frac{16\sqrt{2}}{3\sqrt{3}} Y^2 \right) \sqrt{2} M_1 M_2 E_3^{(2)}. \quad (49)$$

Looking at the spin-1 case we find that the coefficient  $U_2^{(1)}$  is sensitive to  $E_1^{(1)}$ . And hence this term can be used to identify



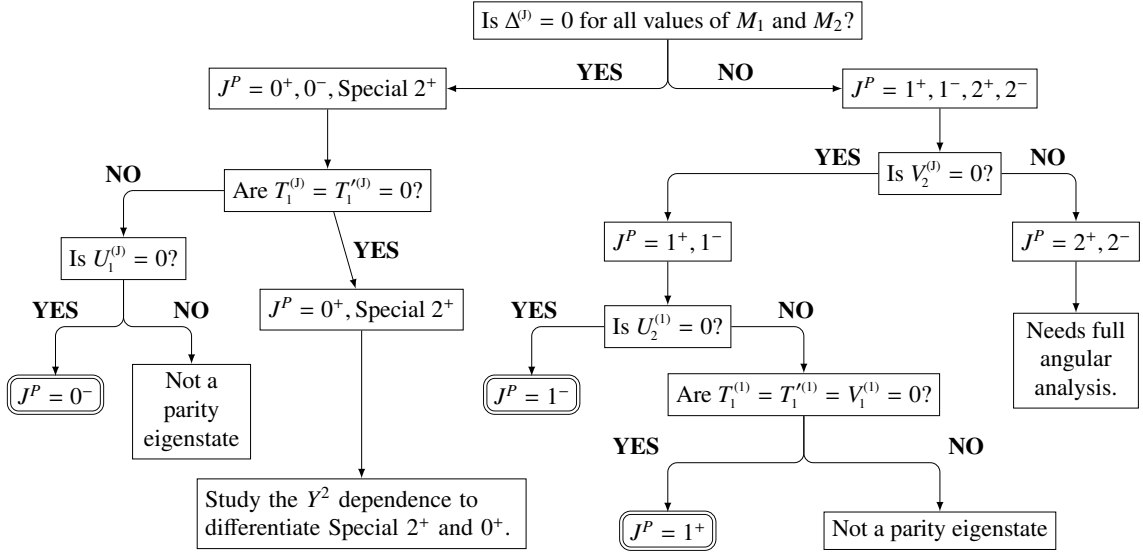


FIG. 2. Flowchart to determine the spin and parity of a resonance  $X$  decaying as  $X \rightarrow Z_1 Z_2 \rightarrow (\ell_1^- \ell_1^+)(\ell_2^- \ell_2^+)$ .

the parity of the spin-1 resonance. We can thus find out a step-by-step methodology that can be followed to uniquely predict the spin and parity of a resonance decaying to four final charged leptons via two  $Z$  bosons. This is given in Fig. 2.

### III. NUMERICAL STUDY

In this section we will show how the uniangular distributions given in Eqs. (26), (27) and (28) can be used to find out the values of the angular observables defined in Eqs. (29), (30), (31), (32), (33), (34), (35) and (36). We shall elucidate the methodology by concentrating on some heavy spin 1 resonances and study the observables for them. We start by investigating how the mass and decay width of such resonances affect their production cross-section in the future LHC runs. We then benchmark the angular observables for spin-1<sup>+</sup> and spin-1<sup>-</sup> resonances for two different Center-of-Momentum (CM) energies: 14 TeV and 33 TeV with 3000 fb<sup>-1</sup> luminosity. This analysis can be easily extended to consider a heavy spin-2 resonance.

Let us consider a heavy spin-1 resonance  $X$  of mass  $M_X$  and decay width  $\Gamma_X$ , decaying into four charged leptons via two  $Z$  bosons. We shall assume that the resonance  $X$  is produced via annihilation of quark ( $q$ ) and antiquark ( $\bar{q}$ ) pairs. The production process is characterized by the effective Lagrangian,

$$\mathcal{L}_{eff} = \sum_q \left( \tilde{c}_q \bar{q} \gamma^\mu q X_\mu + c_q \bar{q} \gamma^\mu \gamma^5 q X_\mu \right), \quad (50)$$

where  $q = u, d, c, s, b$  quarks and  $\tilde{c}_q, c_q$  are the coupling strengths of  $X$  to vector, axial vector currents respectively; i.e. for a spin 1<sup>+</sup> resonance  $\tilde{c}_q = 0$  and for a spin 1<sup>-</sup> resonance  $c_q = 0$ . Just for simplicity of the analysis we have further assumed that all the quarks couple to the resonance  $X$  with the same strength. The production cross section for the resonance

$X$  can be easily obtained from Eq. (50) by considering appropriate parton distribution functions in the process  $pp \rightarrow X$ . The production cross section does depend on the mass of the resonance  $X$ : larger the mass, lower is the production cross section at a given CM energy.

The decay process  $X \rightarrow ZZ$  is characterized by the Lagrangian Eq. (6). The partial decay width for  $X \rightarrow ZZ$  for spin-1 resonance is given by

$$\Gamma_{ZZ} = O_1^2 \frac{M_X^3}{32 M_Z^2 \pi} \left( 1 - \frac{4 M_Z^2}{M_X^2} \right)^{\frac{3}{2}} + E_1^2 \frac{M_X^3}{32 M_Z^2 \pi} \left( 1 - \frac{4 M_Z^2}{M_X^2} \right)^{\frac{5}{2}}. \quad (51)$$

For a spin-1<sup>+</sup> resonance  $O_1 = 0$  and for a spin-1<sup>-</sup> resonance  $E_1 = 0$ . It must be noted that we have dropped the superscript “(1)” from both  $E_1$  and  $O_1$  throughout this section.

The partial decay width for  $X \rightarrow q\bar{q}$  for spin-1 resonance is given by

$$\Gamma_{q\bar{q}} = c_q^2 \frac{M_X}{4\pi} \left( 1 - \frac{4m_q^2}{M_X^2} \right)^{\frac{3}{2}} + \tilde{c}_q^2 \frac{(M_X^2 + 2m_q^2)}{4 M_X \pi} \left( 1 - \frac{4m_q^2}{M_X^2} \right)^{\frac{1}{2}}, \quad (52)$$

where  $m_q$  is the mass of the quark  $q$  (or of antiquark  $\bar{q}$ ), and  $c_q = 0$  for spin-1<sup>-</sup> resonance and  $\tilde{c}_q = 0$  for spin-1<sup>+</sup> resonance. Let us further assume that  $X$  decays to all quark-antiquark pairs and to a pair of  $Z$  bosons only, i.e. the total decay width is given by

$$\Gamma_X = \Gamma_{ZZ} + \sum_q \Gamma_{q\bar{q}}. \quad (53)$$

One can relax these simple assumptions and do a detailed analysis where other decay channels also exist. This will lead to modifications to the Eq. (53). In Fig. 3 we show how the partial decay width  $\Gamma_{ZZ}$  varies with the mass  $M_X$  of the spin-1<sup>+</sup> resonance, with  $M_Z \ll M_X$ . The spin-1<sup>-</sup> resonances also exhibit a similar plot for  $M_Z \ll M_X$ .

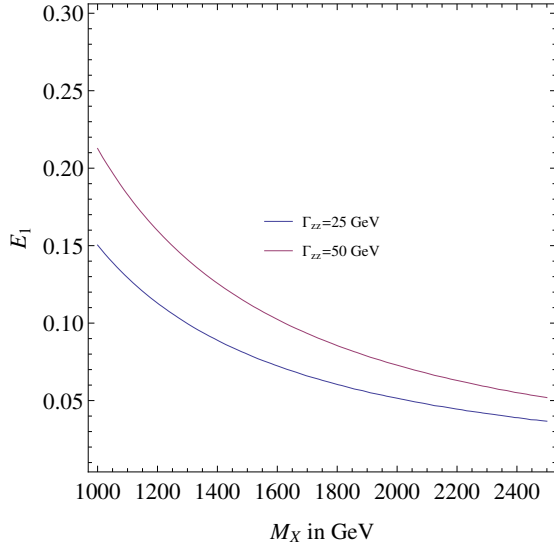


FIG. 3. Mass ( $M_X$ ) vs.  $E_1$  plot of a Spin  $1^+$  resonance for different  $\Gamma_{ZZ}$ . The blue curve for  $\Gamma_{ZZ} = 25$  and the purple curve for  $\Gamma_{ZZ} = 50$ .

One possibility for a heavy spin-1 resonance is a heavy  $Z'$  boson. The current limit on the mass of a heavy  $Z'$  resonance is 1.7 TeV [30]. The current limit of  $\tilde{c}_q$  and  $c_q$  for a particular mass  $M_X$  of the resonance  $X$ , can be extracted out from the  $\sigma \times Br \times \mathcal{A}$  vs. resonance mass ( $M_X$ ) plot of Ref. [30], where  $\sigma$  is the cross-section for the process  $pp \rightarrow X$ ,  $Br$  is the branching fraction of the decay  $X \rightarrow q\bar{q}$  and  $\mathcal{A}$  is the acceptance. Since the analysis of Ref. [30] deals with search for a heavy resonance  $Z'$  decaying to di-jet, which is an isotropic decay (two body final state), the acceptance  $\mathcal{A}$  is approximately 0.6 and is independent of the mass of  $Z'$ .

Following the analysis of Ref. [30] we find the allowed region for the couplings  $c_q$  and  $E_1$  for two different masses,  $M_X = 1.8$  TeV and 2 TeV, shown in Figs. 4 and 5 respectively. From the allowed regions shown in Figs. 4 and 5, we choose three benchmark scenarios for masses  $M_X = 1.8$  TeV and  $M_X = 2.0$  TeV for our numerical study. The benchmark values of  $c_q$  and  $E_1$  corresponding to both the masses are tabulated in Table III. Once the values of  $M_X$ ,  $c_q$  (or  $\tilde{c}_q$ ) and  $E_1$  (or

Mass	Coupling $c_q$	Coupling $E_1$	$\Gamma_X$ in GeV
1.8 TeV	0.12	$7.00 \times 10^{-2}$	64.40
1.8 TeV	0.10	$8.56 \times 10^{-2}$	71.52
2.0 TeV	0.10	$8.56 \times 10^{-2}$	92.84

TABLE III. The benchmark values of the couplings  $c_q$  and  $E_1$ , for a spin- $1^+$  resonance of masses 1.8 TeV and 2 TeV respectively for our analysis. The values of corresponding decay widths  $\Gamma_X$  are also tabulated in the last column for both the masses.

$O_1$ ) for a spin- $1^+$  (or spin- $1^-$ ) resonance are chosen, the total decay width  $\Gamma_X$  as well as the cross-section for the process  $pp \rightarrow X \rightarrow ZZ \rightarrow e^+e^-\mu^+\mu^-$  get fixed. The reader should note that, the process under consideration is within the narrow width approximation where  $\Gamma_X \ll M_X$ .

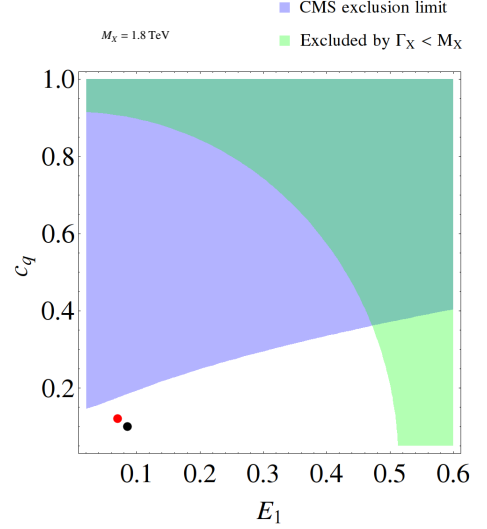


FIG. 4. The allowed region for the couplings  $c_q$  and  $E_1$  for a spin- $1^+$  resonance of mass  $M_X = 1.8$  TeV. The green and the blue regions are excluded by  $\Gamma_X < M_X$  limit and CMS limit from Ref. [30] respectively. The red ( $E_1 = 7.00 \times 10^{-2}$ ,  $c_q = 0.12$ ) and the black ( $E_1 = 8.56 \times 10^{-2}$ ,  $c_q = 0.10$ ) dots are the two benchmark points for our analysis.

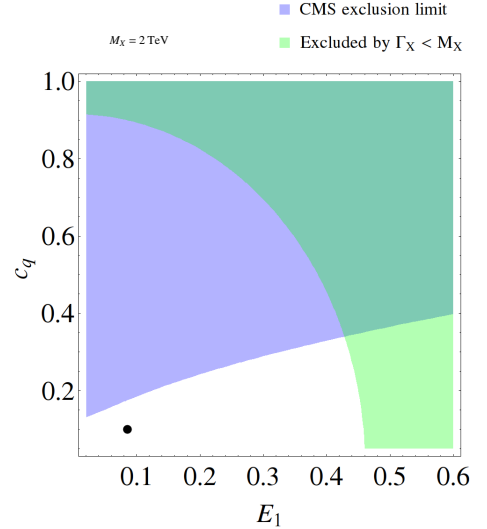


FIG. 5. The allowed region for the couplings  $c_q$  and  $E_1$  for a spin- $1^+$  resonance of mass  $M_X = 2$  TeV. The green and the blue regions are excluded by  $\Gamma_X < M_X$  limit and CMS limit respectively. The black dot ( $E_1 = 8.56 \times 10^{-2}$ ,  $c_q = 0.10$ ) is the benchmark point for our analysis.

For event generations we have used MADEVENT5 [31] event generator interfaced with PYTHIA6.4 [32] and Delphes 3 [33]. The events are generated by  $pp$  collisions via  $q\bar{q} \rightarrow X$ , for the CM energies  $\sqrt{s} = 14$  TeV and 33 TeV, using the parton distribution functions CTEQ6L1 [34]. Triggers as well as electron and muon identification cuts are set following the analysis presented in Refs. [35, 36]. We have only selected events with final states  $e^+e^-\mu^+\mu^-$ , as our analysis is applica-



ble only to four non-identical final state leptons. We have kept the trigger values same as 14 TeV, for the 33 TeV LHC analysis. However, it should be noted that in the future 33 TeV LHC run, the trigger values may change, which could further improve the statistics. The electron (muon) must satisfy  $E_T > 7$  GeV ( $p_T > 6$  GeV) and the pseudo-rapidity cut for electron (muon) is  $|\eta| < 2.47$  ( $|\eta| < 2.7$ ). The leptons are required to be separated from each other by  $\Delta R > 0.1$  if they are of the same flavour and  $\Delta R > 0.2$  otherwise. The invariant mass cuts that are applied in our analysis are  $60 \text{ GeV} < m_{ee} < 120 \text{ GeV}$ ,  $60 \text{ GeV} < m_{\mu\mu} < 120 \text{ GeV}$  and  $1000 \text{ GeV} < m_{4\ell}$ .

The effects of mass  $M_X$  and width  $\Gamma_X$  on  $\sigma \times Br$  are shown in Table IV for a spin- $1^+$  resonance. The statistics lowers down as the resonance gets heavier. However, the statistics improves for a resonance with the same mass but narrower decay width. This dependence is easily discernible in Table IV for a spin- $1^+$  resonance. This mass and width dependence on the cross-sections for the  $pp \rightarrow X \rightarrow ZZ \rightarrow e^+e^-\mu^+\mu^-$  process also shows the same behavior for a spin- $1^-$  resonance in the limit  $m_q \ll M_X$  and  $M_Z \ll M_X$ . So far we have not discussed about the background for the  $pp \rightarrow X \rightarrow e^+e^-\mu^+\mu^-$  process. This is discussed in the following subsection.

#### A. Study of the angular asymmetries for a spin- $1^+$ resonance:

In this subsection we discuss the uniangular distributions and show how to extract the angular observables from them for a spin- $1^+$  resonance of mass  $M_X = 1.8 \text{ TeV}$  and decay width  $\Gamma_X = 64.04 \text{ GeV}$ . We choose this benchmark scenario as the statistics is higher for this than the other cases. We analyse the angular observables for this benchmark scenario for two different CM energies 14 TeV and 33 TeV at an integrated luminosity  $3000 \text{ fb}^{-1}$  in future LHC runs. The values of the couplings for this benchmark scenarios are  $c_q = 0.12$  and  $E_1 = 7.00 \times 10^{-2}$ . The effects of the sequential cuts for the benchmark scenarios are tabulated in Table V. The three uniangular distributions for a spin- $1^+$  resonance,  $\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_1}$  vs.  $\cos \theta_1$ ,  $\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_2}$  vs.  $\cos \theta_2$  and  $\frac{1}{\Gamma} \frac{d\Gamma}{d \phi}$  vs.  $\phi$  are shown in Figs. 6, 7 and 8 respectively. It should be noted that the uniangular distributions cover the full kinematic ranges for the three variables  $\cos \theta_1$ ,  $\cos \theta_2$  and  $\phi$ .

However, while extracting observables one has to take the background processes into account. The  $pp \rightarrow e^+e^-\mu^+\mu^-$  process is a continuum background to the process  $pp \rightarrow ZZ \rightarrow e^+e^-\mu^+\mu^-$ . The effects of the sequential cuts on the background process for 14 TeV and 33 TeV  $3000 \text{ fb}^{-1}$  LHC run, are shown in the Table VI. The simulated signal and background events are finally binned in  $\cos \theta_1$ ,  $\cos \theta_2$  and  $\phi$  and fitted using Eqs. (26), (27) and (28) integrated over  $m_{ee}^2 (\equiv q_1^2)$  and  $m_{\mu\mu}^2 (\equiv q_2^2)$  to obtain the *integrated* angular observables with their respective errors. The values of the observables are tabulated in Table VII for the two different CM energies. It is clear from Table VII that the observable  $T_2$  and  $T'_2$  extracted from  $\cos \theta_1$  and  $\cos \theta_2$  distributions respectively, match within  $1\sigma$  error. This is expected since both  $q_1^2$  and  $q_2^2$  are integrated

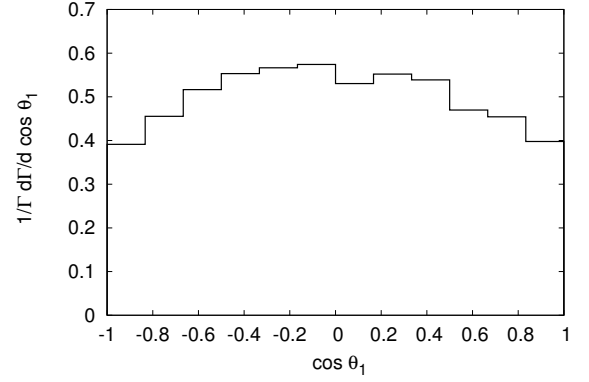


FIG. 6. The normalized distribution  $\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_1}$  vs.  $\cos \theta_1$  for a Spin- $1^+$  resonance of mass  $M_X = 1.8 \text{ TeV}$  and width  $\Gamma_X = 64.04 \text{ GeV}$ .

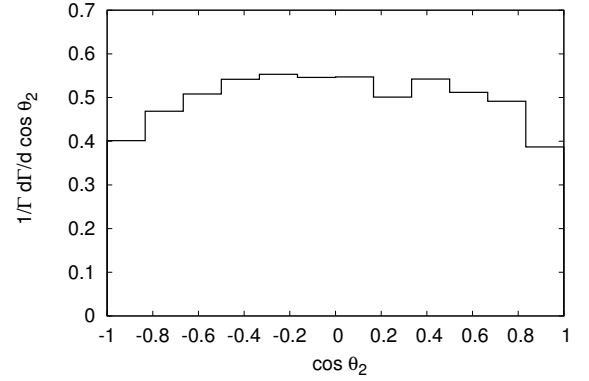


FIG. 7. The normalized distribution  $\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_2}$  vs.  $\cos \theta_2$  for a Spin- $1^+$  resonance of mass  $M_X = 1.8 \text{ TeV}$  and width  $\Gamma_X = 64.04 \text{ GeV}$ .

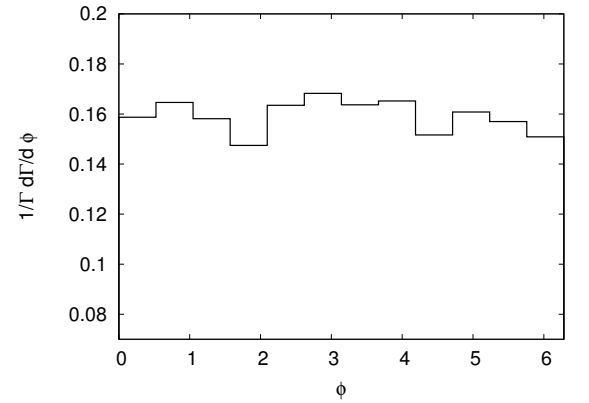


FIG. 8. The normalized distribution  $\frac{1}{\Gamma} \frac{d\Gamma}{d \phi}$  vs.  $\phi$  for a Spin- $1^+$  resonance of mass  $M_X = 1.8 \text{ TeV}$  and width  $\Gamma_X = 64.04 \text{ GeV}$ .

Cuts in GeV	$M_X = 1.8$ TeV, $\Gamma_X = 64.40$ GeV	$M_X = 1.8$ TeV, $\Gamma_X = 71.52$ GeV	$M_X = 2$ TeV, $\Gamma_X = 92.84$ GeV
Selection cuts	231	216	111
$60 < m_{ee} < 120$	231	216	111
$60 < m_{\mu\mu} < 120$	222	208	106
$1000 < m_{4\ell}$	221	207	106

TABLE IV. Effects of the sequential cuts on the simulated events at 14 TeV 3000 fb<sup>-1</sup> LHC for different values of  $M_X$  and  $\Gamma_X$  of a spin-1<sup>+</sup> resonance. It is easy to observe from the benchmark scenarios considered in this table that at a given CM energy the production cross section decreases with increase in  $M_X$  and for a fixed value of  $M_X$  the production cross section decreases as the value of the decay width  $\Gamma_X$  increases.

Cuts in GeV	14 TeV, 3000 fb <sup>-1</sup>	33 TeV, 3000 fb <sup>-1</sup>
Selection cuts	231	1212
$60 < m_{ee} < 120$	231	1212
$60 < m_{\mu\mu} < 120$	222	1159
$1000 < m_{4\ell}$	221	1154

TABLE V. The effects of the sequential cuts on the simulated signal events at 14 TeV and 33 TeV LHC with 3000 fb<sup>-1</sup> luminosity for a spin-1<sup>+</sup> resonance with  $M_X = 1.8$  TeV and width  $\Gamma_X = 64.40$  GeV.

Cuts in GeV	14 TeV, 3000 fb <sup>-1</sup>	33 TeV, 3000 fb <sup>-1</sup>
Selection cuts	24530	48588
$60 < m_{ee} < 120$	23320	46949
$60 < m_{\mu\mu} < 120$	18468	40082
$1000 < m_{4\ell}$	41	238

TABLE VI. The effects of the sequential cuts on the simulated background events at 14 TeV and 33 TeV LHC with 3000 fb<sup>-1</sup> luminosity.

Observables	14 TeV, 3000 fb <sup>-1</sup>	33 TeV, 3000 fb <sup>-1</sup>
$T_2$	$-0.19 \pm 0.11$	$-0.18 \pm 0.06$
$T_1$	$0.07 \pm 0.09$	$0.01 \pm 0.04$
$T'_2$	$-0.09 \pm 0.12$	$-0.10 \pm 0.06$
$T'_1$	$-0.04 \pm 0.10$	$-0.03 \pm 0.05$
$U_2$	$0.08 \pm 0.51$	$0.04 \pm 0.24$
$U_1$	$(-0.87 \pm 5.33) \times 10^{-1}$	$(-0.17 \pm 2.40) \times 10^{-1}$
$V_2$	$(-0.41 \pm 5.32) \times 10^{-1}$	$(0.21 \pm 2.36) \times 10^{-1}$
$V_1$	$(-0.32 \pm 5.11) \times 10^{-1}$	$(0.30 \pm 2.34) \times 10^{-1}$

TABLE VII. The fit values and the respective errors of the observables  $T_1$ ,  $T_2$ ,  $U_1$ ,  $U_2$ ,  $V_1$  and  $V_2$  for a Spin-1<sup>+</sup> resonance of mass  $M_X = 1.8$  TeV and width  $\Gamma_X = 64.04$  GeV at 14 TeV and 33 TeV LHC run (with 3000 fb<sup>-1</sup> luminosity).

over the same range. A full implementation of the flow chart (shown in Fig. 2) will require a fit with at least two regions  $q_1^2 < q_2^2$  and  $q_1^2 > q_2^2$ . However, given the heavy mass for  $X$  the production cross section is low, hence, the errors are still large and more statistics is needed to undertake such a study.

### B. Study of the angular asymmetries for a spin-1<sup>-</sup> resonance

We have so far discussed the possibility of finding a heavy spin-1<sup>+</sup> resonance. However, the resonance may well could be a spin-1<sup>-</sup>. The limits on the couplings  $\tilde{c}_q$  and  $O_1$  can also be found from  $\sigma \times Br \times \mathcal{A}$  limit from Ref. [30]. In the limit  $M_Z \ll M_X$  and  $m_q \ll M_X$ , the couplings  $\tilde{c}_q \approx c_q$  and  $O_1 \approx E_1$ . Hence, we choose the values  $\tilde{c}_q = 0.12$  and  $O_1 = 7.00 \times 10^{-2}$  for the couplings as a benchmark scenario for our analysis of a spin-1<sup>-</sup> resonance of mass 1.8 TeV and decay width  $\Gamma_X = 64.40$  GeV. We perform the same analysis as given in Sec. III A and extract out the values of the angular observables at two different CM energies, 14 TeV and 33 TeV at an integrated luminosity of 3000 fb<sup>-1</sup>. We also find three uniangular distributions for the spin-1<sup>-</sup> resonance, shown in Figs. 9, 10 and 11 respectively. The effects of the sequential cuts are tabulated in Table VIII at 14 TeV and 33 TeV LHC for 3000 fb<sup>-1</sup> luminosity. The background analysis for spin-1<sup>-</sup> resonance would remain the same as stated in Sec III A.

Cuts in GeV	14 TeV, 3000 fb <sup>-1</sup>	33 TeV, 3000 fb <sup>-1</sup>
Selection cuts	220	1155
$60 < m_{ee} < 120$	200	1154
$60 < m_{\mu\mu} < 120$	211	1108
$1000 < m_{4\ell}$	210	1105

TABLE VIII. The effects of the sequential cuts on the simulated signal events at 14 TeV 3000 fb<sup>-1</sup> and 33 TeV 3000 fb<sup>-1</sup> LHC of a spin-1<sup>-</sup> resonance with  $M_X = 1.8$  TeV and width  $\Gamma_X = 64.40$  GeV.

The observables extracted from the uniangular distributions of the spin-1<sup>-</sup> resonance are given in Table IX. Apart from  $T_2$  the errors of the other observables are still not small and require higher statistics to fully study the flowchart 2.

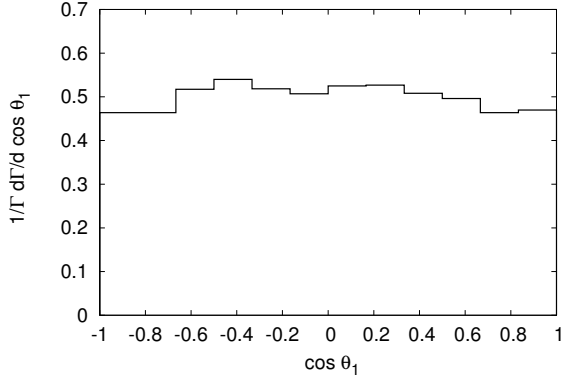


FIG. 9. The normalized distribution  $\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_1}$  vs.  $\cos \theta_1$  for a Spin  $1^-$  resonance of mass  $M_X = 1.8$  TeV and width  $\Gamma_X = 64.04$  GeV.

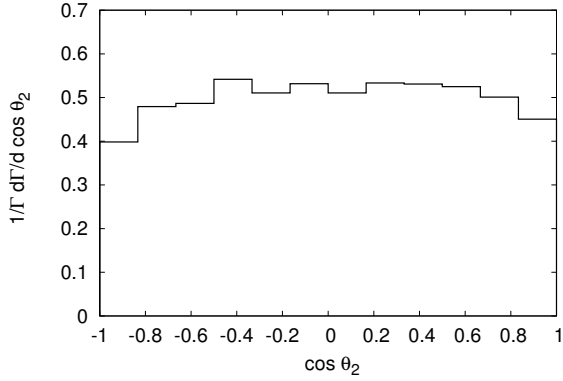


FIG. 10. The normalized distribution  $\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_2}$  vs.  $\cos \theta_2$  for a Spin  $1^-$  resonance of mass  $M_X = 1.8$  TeV and width  $\Gamma_X = 64.04$  GeV.

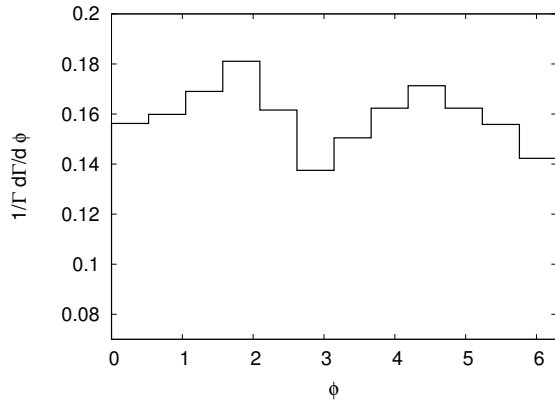


FIG. 11. The normalized distribution  $\frac{1}{\Gamma} \frac{d\Gamma}{d \phi}$  vs.  $\phi$  for a Spin  $1^-$  resonance of mass  $M_X = 1.8$  TeV and width  $\Gamma_X = 64.04$  GeV.

Observables	14 TeV, 3000 fb <sup>-1</sup>	33 TeV, 3000 fb <sup>-1</sup>
$T_2$	$-0.07 \pm 0.13$	$-0.12 \pm 0.06$
$T_1$	$0.06 \pm 0.10$	$0.01 \pm 0.04$
$T'_2$	$-0.04 \pm 0.13$	$-0.07 \pm 0.06$
$T'_1$	$0.09 \pm 0.10$	$-0.01 \pm 0.04$
$U_2$	$-0.08 \pm 0.54$	$-0.05 \pm 0.24$
$U_1$	$(1.99 \pm 5.16) \times 10^{-1}$	$(1.20 \pm 2.32) \times 10^{-1}$
$V_2$	$(-0.14 \pm 5.27) \times 10^{-1}$	$(0.58 \pm 2.33) \times 10^{-1}$
$V_1$	$(-0.27 \pm 5.46) \times 10^{-1}$	$(-0.63 \pm 2.39) \times 10^{-1}$

TABLE IX. The fit values and the errors of the observables  $T_1$ ,  $T_2$ ,  $U_1$ ,  $U_2$ ,  $V_1$  and  $V_2$  for a Spin  $1^-$  resonance of mass  $M_X = 1.8$  TeV and width  $\Gamma_X = 64.04$  GeV for signal plus background. The values are extracted for two different CM energies, 14 TeV and 33 TeV LHC run with luminosity 3000 fb<sup>-1</sup>.

If a heavy spin-1 resonance is seen at the LHC, the full angular analysis and the extraction of all the observables may not be entirely possible at 33 TeV 3000 fb<sup>-1</sup> run. Once such resonance is observed a future high luminous machine could disentangle the exact spin and parity of the resonance studying the observables extracted from uniangular distributions. Moreover, we have not discussed the spin-1 resonance with mixed parity configuration and would typically require higher statistics as well, to completely disentangle, which is beyond the scope of the current paper.

#### IV. CONCLUSION

We conclude that by looking at three *normalized* uniangular distributions, for the decay of a resonance to four charged final leptons via two  $Z$  bosons, one can infer to a fairly good accuracy the spin and parity of the parent particle. We show that it is possible for a *special*  $2^+$  resonance to give angular distributions comparable to those of a  $0^+$  resonance. It is in this special case that one needs to study the  $Y^2$  dependence of the helicity amplitudes in order to distinguish the two cases. Since the spin-1 case has only two helicity amplitudes, it needs minimum number of observables to get confirmed or ruled out. We have also provided a step-by-step methodology that must be followed to distinguish the various spin, parity possibilities that are allowed in the case under consideration. A numerical analysis has also been performed, for a heavy spin 1 resonance to validate our formalism. It would therefore be not an overstatement to say that this method can play a crucial role, at future high luminosity machines, in discovering the spin-parity nature of any new resonance, such as a heavy scalar boson or a  $Z'$  boson or a Kaluza-Klein boson or any such resonance, found to decay to four final charged leptons via two  $Z$  bosons.

## ACKNOWLEDGMENTS

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## Appendix A: Relationships amongst the form factors and the effective coupling constants

The form factors  $E_i^{(j)}, O_i^{(j)}$  (which enter the vertex factors in Eqs. (12), (13) and (14)) are related to the effective coupling constants  $e_i^{(j)}, o_i^{(j)}$  (which enter the Lagrangians given in Eqs. (5), (6) and (7)) in the following manner

$$E_1^{(0)} = e_1^{(0)} + 4e_2^{(0)}(q_1 \cdot q_2), \quad (\text{A1})$$

$$E_2^{(0)} = -4e_2^{(0)}, \quad (\text{A2})$$

$$O_1^{(0)} = -8o_1^{(0)}, \quad (\text{A3})$$

$$O_1^{(1)} = -o_1^{(1)}, \quad (\text{A4})$$

$$E_1^{(1)} = 2e_1^{(1)}, \quad (\text{A5})$$

$$E_1^{(2)} = e_1^{(2)} + \frac{1}{4}e_2^{(2)}(P^2 - Q^2) + \frac{1}{16}e_3^{(2)}(P^2 - Q^2)^2 + \frac{1}{4}e_4^{(2)}(P^4 - (P \cdot Q)^2), \quad (\text{A6})$$

$$E_2^{(2)} = \frac{1}{4}(4e_2^{(2)} + e_3^{(2)}(P^2 - Q^2) + 2e_4^{(2)}(P^2 + P \cdot Q)), \quad (\text{A7})$$

$$E_3^{(2)} = \frac{1}{4}(-2e_2^{(2)} - e_5^{(2)}(P^2 - Q^2)), \quad (\text{A8})$$

$$E_4^{(2)} = \frac{1}{2}(e_3^{(2)} + e_4^{(2)} - 2e_5^{(2)}), \quad (\text{A9})$$

$$O_1^{(2)} = 4o_1^{(2)} + o_2^{(2)}(P^2 - Q^2) + 2o_3^{(2)}(P^2 + P \cdot Q), \quad (\text{A10})$$

$$O_2^{(2)} = o_2^{(2)} + o_3^{(2)}, \quad (\text{A11})$$

$$O_3^{(2)} = -o_1^{(2)}, \quad (\text{A12})$$

$$O_4^{(2)} = o_4^{(2)}. \quad (\text{A13})$$

## Appendix B: Helicity amplitudes and partial wave contributions for the decay $X \rightarrow ZZ$

If we specify the polarizations of the initial and final particles, then the Feynman amplitude or transition amplitude can always be written in terms of helicity amplitudes. We shall represent the polarisation state of a particle by a ket  $|\text{spin}, \text{spin projection to } z \text{ axis}\rangle$ . Then the Feynman amplitude for the process

$$\underbrace{|\mathbf{J}, J_z\rangle}_X \rightarrow \underbrace{|\mathbf{1}, \lambda_1\rangle}_{Z_1} \underbrace{|\mathbf{1}, \lambda_2\rangle}_{Z_2}$$

is given by the well known expression [37, 38] involving the Wigner- $D$  function  $\mathcal{D}_{J_z, \lambda}^{J*}(\phi, \theta, -\phi)$ :

$$\mathcal{M}(J_z, \lambda_1, \lambda_2) = \left(\frac{2J+1}{4\pi}\right)^{\frac{1}{2}} \mathcal{D}_{J_z, \lambda}^{J*}(\phi, \theta, -\phi) A_{\lambda_1, \lambda_2}, \quad (\text{B1})$$

where  $\lambda = |\lambda_1 - \lambda_2|$  with  $\lambda_{1,2} \in \{\pm 1, 0\}$ ,  $J = |\mathbf{J}|$ , and  $A_{\lambda_1, \lambda_2}$  is called the *helicity amplitude*. Conservation of angular momentum implies that

$$|\lambda| = |\lambda_1 - \lambda_2| \leq J. \quad (\text{B2})$$

Since there are no interferences amongst the amplitudes with different helicity configurations, we will have to sum over all the allowed values of  $\lambda_1$  and  $\lambda_2$  that are not constrained by the value of  $J_z$  after squaring each individual amplitude:

$$|\mathcal{M}|^2 = \sum_{\substack{\lambda_1, \lambda_2 \\ |\lambda_1 - \lambda_2| \leq J}} |\mathcal{M}(J_z, \lambda_1, \lambda_2)|^2 = \left(\frac{2J+1}{4\pi}\right) \sum_{\substack{\lambda_1, \lambda_2 \\ |\lambda_1 - \lambda_2| \leq J}} |\mathcal{D}_{J_z, \lambda}^{J*}(\phi, \theta, -\phi)|^2 |A_{\lambda_1, \lambda_2}|^2. \quad (\text{B3})$$

Thus the probability of contribution of the helicity amplitude  $A_{\lambda_1, \lambda_2}$  to the transition amplitude can be found as  $\mathcal{M}(J_z, \lambda_1, \lambda_2)$  is  $\left(\frac{2J+1}{4\pi}\right) |\mathcal{D}_{J_z, \lambda}^{J*}(\phi, \theta, -\phi)|^2$ . We can therefore write down the following important fact of the helicity amplitude formalism: *All the allowed helicity amplitudes for a given decay process contribute, but with different definite probability, to the Feynman amplitude, irrespective of the polarization of the parent (decaying) particle. The probability, however, depends on the polarization of the parent particle and for all allowed helicity amplitudes is non-zero.* Since the two  $Z$  bosons are Bose symmetric, the helicity amplitudes satisfy the relation

$$A_{\lambda_2, \lambda_1} = (-1)^J A_{\lambda_1, \lambda_2} = \begin{cases} +A_{\lambda_1, \lambda_2} & \text{for spin-0, 2} \\ -A_{\lambda_1, \lambda_2} & \text{for spin-1} \end{cases}. \quad (\text{B4})$$

This relationship is useful in getting the correct number of independent helicity amplitudes. All the allowed helicity amplitudes in the decay  $X \rightarrow ZZ$  are given in Table X where  $N$  denotes the total number independent helicity amplitudes possible for the particular spin case.

Spin of $X$	Allowed Helicity Amplitudes	$N$
0	$A_{++}, A_{00}, A_{--}$	3
1	$A_{+0} = -A_{0+}, A_{0-} = -A_{-0}$	2
2	$A_{++}, A_{00}, A_{--}, A_{+-} = A_{-+}, A_{+0} = A_{0+}, A_{0-} = A_{-0}$	6

TABLE X. Allowed helicity amplitudes considering only the different spin possibilities.

It is also known that, if the particle  $X$  were a parity eigenstate with eigenvalue  $\eta_X = +1$  (parity-even) or  $-1$  (parity-odd), then the helicity amplitudes are related by:

$$A_{\lambda_1, \lambda_2} = \eta_X (-1)^J A_{-\lambda_1, -\lambda_2}. \quad (\text{B5})$$

The allowed helicity amplitudes for the different spin-parity possibilities can thus be related and are given in Table XI. It is clearly evident from above that for the spin-0 case out

$J^P$ of $X$	Allowed Helicity Amplitudes	N
$0^+$	$A_{++} = A_{--}, A_{00}$	2
$0^-$	$A_{++} = -A_{--}$	1
$1^+$	$A_{+0} = -A_{-0} = A_{0-} = -A_{0+}$	1
$1^-$	$A_{+0} = A_{-0} = -A_{0-} = -A_{0+}$	1
$2^+$	$A_{++} = A_{--}, A_{00}, A_{+-} = A_{-+},$ $A_{+0} = A_{-0} = A_{0-} = A_{0+}$	4
$2^-$	$A_{++} = -A_{--},$ $A_{+0} = -A_{-0} = -A_{0-} = A_{0+}$	2

TABLE XI. Relationships amongst the allowed helicity amplitudes for the different spin-parity cases.

of the three helicity amplitudes two describe the parity-even scenario and only one describes the parity-odd scenario. Similarly, for spin-1 both parity-even and parity-odd cases are described by one helicity amplitude each. For spin-2 case, we have four helicity amplitudes describing the parity-even scenario and two helicity amplitudes for the parity-odd scenario.

If we now make a change of basis from the helicity basis to the transversity basis (also called as linear polarization basis), then the number of independent transversity amplitudes must be equal to the number of helicity amplitudes, and *all the allowed transversity amplitudes would contribute with definite probability to the Feynman amplitude irrespective of the polarization of the parent (decaying) particle.*

Let us now analyse the decay process from the point-of-view of partial wave decompositions. If we describe the two  $Z$  boson system by a ket specifying the total spin ( $\mathbf{L}_{\text{spin}}$ ), the relative orbital angular momentum ( $\mathbf{L}_{\text{orbital}}$ ), the spin of the parent particle (its  $\mathbf{J}$  here) and its projection along the direction of flight of one of the  $Z$  bosons ( $J_z$ ):  $|\mathbf{J}, J_z; \mathbf{L}_{\text{orbital}}, \mathbf{L}_{\text{spin}}\rangle$ , then

$$\begin{aligned} & \hat{P}_{12} |\mathbf{J}, J_z; \mathbf{L}_{\text{orbital}}, \mathbf{L}_{\text{spin}}\rangle \\ &= (-1)^{L_{\text{orbital}} + L_{\text{spin}}} |\mathbf{J}, J_z; \mathbf{L}_{\text{orbital}}, \mathbf{L}_{\text{spin}}\rangle, \end{aligned} \quad (\text{B6})$$

where  $\hat{P}_{12}$  is the operator that exchanges the two  $Z$  bosons (it exchanges both their momenta and spins or polarisations),  $L_{\text{orbital}}$  and  $L_{\text{spin}}$  are the modulus of  $\mathbf{L}_{\text{orbital}}$  and  $\mathbf{L}_{\text{spin}}$  respectively. It is obvious that for Bose symmetry to be satisfied  $L_{\text{orbital}} + L_{\text{spin}}$  must be even. The allowed partial waves for the decay  $X \rightarrow ZZ$  are enunciated in Table. XII.

It is easy to observe that when  $X$  has spin-0, then there are three helicity amplitudes and three partial wave contributions (one  $S$ -wave, one  $P$ -wave and one  $D$ -wave). When  $X$  has spin-1, then there are only two independent helicity amplitudes and two partial wave contributions (one  $P$ -wave and one  $D$ -wave). Finally when  $X$  has spin-2, then there are six independent helicity amplitudes and six partial wave contributions (one  $S$ -wave, one  $P$ -wave, two  $D$ -waves, one  $F$ -wave

$\mathbf{L}_{\text{spin}}$	$\mathbf{L}_{\text{orbital}}$	$\mathbf{J}$	Partial wave
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$S$ -wave
$\mathbf{0}$	$\mathbf{2}$	$\mathbf{2}$	$D$ -wave
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}, \mathbf{1}, \mathbf{0}$	$P$ -wave
$\mathbf{1}$	$\mathbf{3}$	$\mathbf{2}$	$F$ -wave
$\mathbf{2}$	$\mathbf{0}$	$\mathbf{2}$	$S$ -wave
$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}, \mathbf{1}, \mathbf{0}$	$D$ -wave
$\mathbf{2}$	$\mathbf{4}$	$\mathbf{2}$	$G$ -wave

TABLE XII. Allowed partial waves for all the spin considerations.

and one  $G$ -wave). It is interesting to note that for the spin-0 case the vertex factor has three form factors, for spin-1 case there are two form factors. However, for the spin-2 case we have eight form factors in the vertex factor instead of six. So one needs to consider only six form factors out of which four should be of parity-even nature and two should be of parity-odd nature. Out of the four parity-odd form factors, only  $O_1^{(2)}$  and  $O_2^{(2)}$  contribute to the vertex factor as explained in C.

### Appendix C: Redundancy of inclusion of $O_3^{(2)}, O_4^{(2)}$ in the spin-2 vertex factor

If we include the  $O_3^{(2)}$  term in the vertex factor for spin-2 case, then only the two helicity amplitudes  $A_{01}^{(2)}$  and  $A_{02}^{(2)}$  get modified as follows:

$$A_{01}^{(2)} = \frac{4Y}{3M_+} \left( (O_1^{(2)} - O_3^{(2)}) (M_-^4 - M_X^2 M_+^2) + O_2^{(2)} (4M_+^2 M_X^2 Y^2) \right), \quad (\text{C1})$$

$$A_{02}^{(2)} = \frac{8M_1 M_2 \mu^2 Y}{3\sqrt{3}M_+} (O_1^{(2)} - O_3^{(2)}), \quad (\text{C2})$$

where the previous  $O_1^{(2)}$  is now replaced by  $(O_1^{(2)} - O_3^{(2)})$ . Note that only this combination of the two form factors would be accessible to any experiment. Moreover, all other helicity amplitudes remain unchanged. In the vertex factor of Eq. (14) the term which is proportional to  $O_4^{(2)}$  can be rewritten using Schouten identity as follows:

$$\begin{aligned} & \epsilon^{\alpha\beta\rho\sigma} Q^\mu Q^\nu q_{1\rho} q_{2\sigma} \\ &= \frac{1}{2} \left[ Q^\nu (\epsilon^{\alpha\mu\rho\sigma} Q^\beta - \epsilon^{\beta\mu\rho\sigma} Q^\alpha) \right. \\ & \quad \left. + Q^\mu (\epsilon^{\alpha\nu\rho\sigma} Q^\beta - \epsilon^{\beta\nu\rho\sigma} Q^\alpha) \right] q_{1\rho} q_{2\sigma} \\ & \quad - \frac{Q^2}{4} (\epsilon^{\alpha\beta\mu\rho} P_\rho Q^\nu + \epsilon^{\alpha\beta\nu\rho} P_\rho Q^\mu) \\ & \quad + \frac{P \cdot Q}{4} (\epsilon^{\alpha\beta\mu\sigma} Q^\nu Q_\sigma + \epsilon^{\alpha\beta\nu\sigma} Q^\mu Q_\sigma). \end{aligned} \quad (\text{C3})$$

Thus the form factor  $O_4^{(2)}$  can be absorbed into the existing form factors  $O_2^{(2)}$  and  $O_3^{(2)}$  by redefining them as follows:

$$O_2^{(2)} \rightarrow O_2^{(2)} + \frac{1}{2} O_4^{(2)}, \quad (C4)$$

$$O_3^{(2)} \rightarrow O_3^{(2)} - \frac{Q^2}{4} O_4^{(2)}. \quad (C5)$$

The remaining contribution of  $O_4^{(2)}$  is proportional to  $P \cdot Q$ . Since  $P \cdot Q$  is not Bose symmetric, this term does not contribute to our vertex factor. Therefore the effect of including the  $O_3^{(2)}$  and  $O_4^{(2)}$  terms in the vertex factor can simply be taken care of by the following redefinitions of  $O_1^{(2)}$  and  $O_2^{(2)}$  as follows:

$$O_1^{(2)} \rightarrow O_1^{(2)} - O_3^{(2)} + \frac{Q^2}{4} O_4^{(2)}, \quad (C6)$$

$$O_2^{(2)} \rightarrow O_2^{(2)} + \frac{1}{2} O_4^{(2)}. \quad (C7)$$

Since we cannot have more than six helicity amplitudes for the spin-2 case, and we already have six form factors in the vertex factor, any additional form factor that we introduce in the vertex factor must come in association with the existing form factors, as proved here.

#### Appendix D: Expressions for $T_i^{(J)}$ , $T_i'^{(J)}$ , $U_i^{(J)}$ , $V_i^{(J)}$

The expressions for the coefficients  $T_i^{(J)}$ ,  $T_i'^{(J)}$ ,  $U_i^{(J)}$ ,  $V_i^{(J)}$  that entered the uni-angular distributions are given as follows:

$$T_1^{(0)} = -T_1'^{(0)} = -\frac{3}{2} \eta \operatorname{Re} (F_{E2}^{(0)} F_{O1}^{(0)*}), \quad (D1)$$

$$T_2^{(0)} = T_2'^{(0)} = \frac{1}{4} (1 - 3 |F_{E1}^{(0)}|^2), \quad (D2)$$

$$U_1^{(0)} = -\frac{9\pi^2}{32\sqrt{2}} \eta^2 \operatorname{Re} (F_{E1}^{(0)} F_{E2}^{(0)*}), \quad (D3)$$

$$U_2^{(0)} = \frac{1}{4} (|F_{E2}^{(0)}|^2 - |F_{O1}^{(0)}|^2), \quad (D4)$$

$$V_1^{(0)} = -\frac{9\pi^2}{32\sqrt{2}} \eta^2 \operatorname{Im} (F_{E1}^{(0)} F_{O1}^{(0)*}), \quad (D5)$$

$$V_2^{(0)} = \frac{1}{2} \operatorname{Im} (F_{E2}^{(0)} F_{O1}^{(0)*}), \quad (D6)$$

$$T_1^{(1)} = -\frac{6\sqrt{2}\eta M_X^3 M_1^2 Y}{D_1 D_2} (M_X^2 - M_1^2 + 3M_2^2) \operatorname{Re} (F_{E1}^{(1)} F_{O1}^{(1)*}), \quad (D7)$$

$$T_1'^{(1)} = \frac{6\sqrt{2}\eta M_X^3 M_2^2 Y}{D_1 D_2} (M_X^2 + 3M_1^2 - M_2^2) \operatorname{Re} (F_{E1}^{(1)} F_{O1}^{(1)*}), \quad (D8)$$

$$T_2^{(1)} = -2M_X^2 Y^2 \left( \frac{|F_{O1}^{(1)}|^2}{D_1^2} \left( (M_X^2 - M_1^2)(M_1^2 + 4M_2^2) + 2M_2^4 \right) + \frac{2|F_{E1}^{(1)}|^2}{D_2^2} \left( M_X^2 (M_1^2 + 16M_2^2) - M_2^2 (20M_1^2 - 3M_2^2) \right) \right), \quad (D9)$$

$$T_2'^{(1)} = -2M_X^2 Y^2 \left( \frac{|F_{O1}^{(1)}|^2}{D_1^2} \left( (M_X^2 - M_2^2)(4M_1^2 + M_2^2) + 2M_1^4 \right) + \frac{2|F_{E1}^{(1)}|^2}{D_2^2} \left( M_X^2 (16M_1^2 + M_2^2) - M_1^2 (20M_2^2 - 3M_1^2) \right) \right), \quad (D10)$$

$$U_1^{(1)} = \frac{9\pi^2 \eta^2 M_X^2 M_1 M_2}{16D_1^2 D_2^2} \left( 4|F_{O1}^{(1)}|^2 M_X^2 Y^2 (16M_X^6 M_+^2 + M_X^4 (56M_+^4 - 85M_-^4) + M_X^2 (96M_+^2 M_-^4 - 86M_+^6) - 35M_+^4 M_-^4 + 38M_-^8) - |F_{E1}^{(1)}|^2 \left( (M_X^2 - 2M_+^2)^2 - M_-^4 \right) (4M_X^6 M_+^2 - M_X^4 (5M_+^4 + M_-^4) + 12M_X^2 M_+^2 (M_+^4 - M_-^4) + 3M_+^4 M_-^4 - M_-^8) \right), \quad (D11)$$

$$U_2^{(1)} = -\frac{8M_1^2 M_2^2 M_-^4}{D_2^2} |F_{E1}^{(1)}|^2, \quad (D12)$$

$$V_1^{(1)} = -\frac{9\pi^2 \eta^2 M_X M_1 M_2 Y}{2\sqrt{2} D_1 D_2} (M_X^4 - 2M_X^2 M_+^2 - M_-^4) \times \operatorname{Im} (F_{E1}^{(1)} F_{O1}^{(1)*}), \quad (D13)$$

$$V_2^{(1)} = 0, \quad (D14)$$

$$T_1^{(2)} = -\frac{3\eta}{2M_+^2 \mu^2 \nu^2} \left( 2M_X M_+^3 \nu^2 \operatorname{Re} (F_{E2}^{(2)} F_{O2}^{(2)*}) + M_1^2 \mu^2 \nu^2 \operatorname{Re} (F_{E3}^{(2)} F_{O1}^{(2)*}) + M_2 M_-^2 \times \left( \sqrt{3} M_1 \nu^2 \operatorname{Re} (F_{E3}^{(2)} F_{O2}^{(2)*}) + M_1 \mu^2 \operatorname{Re} (F_{E4}^{(2)} F_{O1}^{(2)*}) + \sqrt{3} M_2 M_-^2 \operatorname{Re} (F_{E4}^{(2)} F_{O2}^{(2)*}) \right) \right), \quad (D15)$$

$$T_1'^{(2)} = \frac{3\eta}{2M_+^2 \mu^2 \nu^2} \left( 2M_X M_+^3 \nu^2 \operatorname{Re} (F_{E2}^{(2)} F_{O2}^{(2)*}) + M_2^2 \mu^2 \nu^2 \operatorname{Re} (F_{E3}^{(2)} F_{O1}^{(2)*}) + M_1 M_-^2 \times \left( -\sqrt{3} M_2 \nu^2 \operatorname{Re} (F_{E3}^{(2)} F_{O2}^{(2)*}) - M_2 \mu^2 \operatorname{Re} (F_{E4}^{(2)} F_{O1}^{(2)*}) + \left( \sqrt{3} M_1^2 M_-^4 \operatorname{Re} (F_{E4}^{(2)} F_{O2}^{(2)*}) \right) \right) \right), \quad (D16)$$

$$T_2^{(2)} = \frac{1}{4} \left( -2|F_{E1}^{(2)}|^2 + |F_{E2}^{(2)}|^2 + (|F_{E3}^{(2)}|^2 + |F_{O1}^{(2)}|^2) \left( \frac{M_1^2 - 2M_2^2}{M_+^2} \right) + |F_{E4}^{(2)}|^2 \left( 2M_X^2 \frac{M_+^2}{\nu^4} + \frac{M_-^4}{M_+^2 \nu^4} (M_2^2 - 2M_1^2) \right) + |F_{O2}^{(2)}|^2 \left( 4M_X^2 \frac{M_+^2}{\mu^4} + 3 \frac{M_-^4}{M_+^2 \mu^4} (M_2^2 - 2M_1^2) \right) + \frac{6M_1 M_2 M_-^2}{M_+^2 \mu^2 \nu^2} \left( \mu^2 \operatorname{Re} (F_{E3}^{(2)} F_{E4}^{(2)*}) + \sqrt{3} \nu^2 \operatorname{Re} (F_{O1}^{(2)} F_{O2}^{(2)*}) \right) \right), \quad (D17)$$

$$T_2'^{(2)} = \frac{1}{4} \left( -2|F_{E1}^{(2)}|^2 + |F_{E2}^{(2)}|^2 + (|F_{E3}^{(2)}|^2 + |F_{O1}^{(2)}|^2) \left( \frac{M_2^2 - 2M_1^2}{M_+^2} \right) \right)$$



$$\begin{aligned}
& + |F_{E4}^{(2)}|^2 \left( 2M_X^2 \frac{M_+^2}{v^4} + \frac{M_-^4}{M_+^2 v^4} (M_1^2 - 2M_2^2) \right) \\
& + |F_{O2}^{(2)}|^2 \left( 4M_X^2 \frac{M_+^2}{\mu^4} + 3 \frac{M_-^4}{M_+^2 \mu^4} (M_1^2 - 2M_2^2) \right) \\
& - \frac{6M_1 M_2 M_-^2}{M_+^2 \mu^2 v^2} \left( \mu^2 \text{Re} \left( F_{E3}^{(2)} F_{E4}^{(2)*} \right) + \sqrt{3} v^2 \text{Re} \left( F_{O1}^{(2)} F_{O2}^{(2)*} \right) \right),
\end{aligned} \tag{D18}$$

$$\begin{aligned}
U_1^{(2)} &= \frac{9\pi^2 \eta^2}{64M_+^2 \mu^4 v^4} \left( \sqrt{2} M_+^2 \mu^4 v^4 \text{Re} \left( F_{E1}^{(2)} F_{E2}^{(2)*} \right) \right. \\
& - M_-^4 \mu^4 v^2 \text{Re} \left( F_{E3}^{(2)} F_{E4}^{(2)*} \right) + |F_{E3}^{(2)}|^2 M_1 M_2 \mu^4 v^4 \\
& - |F_{E4}^{(2)}|^2 M_1 M_2 M_-^4 \mu^4 + \sqrt{3} M_-^4 \mu^2 v^4 \text{Re} \left( F_{O1}^{(2)} F_{O2}^{(2)*} \right) \\
& \left. - |F_{O1}^{(2)}|^2 M_1 M_2 \mu^4 v^4 + 3 |F_{O2}^{(2)}|^2 M_1 M_2 M_-^4 v^4 \right),
\end{aligned} \tag{D19}$$

$$U_2^{(2)} = \frac{1}{4} |F_{E2}^{(2)}|^2 - \frac{M_X^2 M_+^2}{\mu^4} |F_{O2}^{(2)}|^2, \tag{D20}$$

$$\begin{aligned}
V_1^{(2)} &= \frac{9\pi^2 \eta^2}{64M_+^2 \mu^2 v^2} \left( 2\sqrt{2} M_X M_+^3 v^2 \text{Im} \left( F_{E1}^{(2)} F_{O2}^{(2)*} \right) \right. \\
& + 2M_1 M_2 \mu^2 v^2 \text{Im} \left( F_{E3}^{(2)} F_{O1}^{(2)*} \right) - \sqrt{3} M_-^4 v^2 \text{Im} \left( F_{E3}^{(2)} F_{O2}^{(2)*} \right) \\
& \left. - M_-^4 \mu^2 \text{Im} \left( F_{E4}^{(2)} F_{O1}^{(2)*} \right) - 2\sqrt{3} M_-^4 M_1 M_2 \text{Im} \left( F_{E4}^{(2)} F_{O2}^{(2)*} \right) \right),
\end{aligned} \tag{D21}$$

$$V_2^{(2)} = M_X \frac{M_+}{\mu^2} \text{Im} \left( F_{E2}^{(2)} F_{O2}^{(2)*} \right). \tag{D22}$$

Here  $\eta$  is defined as

$$\eta = \frac{2 v_\ell a_\ell}{v_\ell^2 + a_\ell^2}, \tag{D23}$$

with  $v_\ell = 2I_{3\ell} - 4e_\ell \sin^2 \theta_W$  and  $a_\ell = 2I_{3\ell}$ . In the present case  $v_\ell = -1 + 4 \sin^2 \theta_W$  and  $a_\ell = -1$ . Substituting the value  $\sin^2 \theta_W = 0.231$ , we get  $\eta = 0.151$ .

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